Proof from pp 6-7 of book (incorrect in book)

Show for \( N \geq 1 \), \( \sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6} \)

Base Case:
True for \( N = 1 \)
\[
\sum_{i=1}^{N} i^2 = 1^2 = \frac{N(2)(2N+1)}{6} = \frac{2 \times 3}{6} = 1
\]

Inductive Hypothesis
The theorem is true for \( 1 \leq N \leq K \).
Thus, \( \sum_{i=1}^{K} i^2 = \frac{K(K+1)(2K+1)}{6} \)

Show that the theorem is true for \( K+1 \)

\[
\sum_{i=1}^{K+1} i^2 = \sum_{i=1}^{K} i^2 + (K+1)^2 = \frac{K(K+1)(2K+1)}{6} + (K+1)^2
\]

Show \( \frac{K(K+1)(2K+1)}{6} + (K+1)^2 = \frac{(K+1)(2K+1)(2K+3)}{6} \)

Get this by substituting \( K+1 \) in

\[
\frac{K(K+1)(2K+1)}{6} + (K+1)^2 = \frac{(K+1)(2K+1)(2K+3)}{6}
\]

\[
(K+1) \left[ \frac{K(2K+1)}{6} + \frac{6(K+1)}{6} \right]
\]

factor out \( K+1 \)
\[
= (k+1) \left[ \frac{2k^2 + k + 6}{6} \right]
\]
\[
= (k+1) \left[ \frac{2k^2 + 7k + 6}{6} \right]
\]
\[
= (k+1) \left[ \frac{(k+2)(2k+3)}{6} \right]
\]
\[
= \text{factor}
\]
\[
= \frac{(k+1)(k+2)(2k+3)}{6}
\]
\[
= (k+1) \left[ \frac{(k+1)+1}{6} \right]
\]